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A Fluid-Mechanic-Based Model for the Sedimentation of Flocculated Suspensions

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Abstract

A fluid-mechanic-based analysis is presented for the settling behavior of flocculated suspensions. Flocs have been modeled as composite spheres consisting of a solid core embedded in a shell of homogeneous and isotropic porous medium. Theoretical estimates of the rates of sedimentation for flocculated suspensions are obtained by solving the equations of continuity and of motion. The interparticle interactions are incorporated into the analysis by employing the Happel free surface cell model. The results reported embrace wide ranges of conditions of floc size and concentration.

INTRODUCTION

Due to the wide occurrence of the suspensions of fine particles in mineral and chemical processing industries, considerable interest has been shown in modeling the hydrodynamic behavior of such systems. Much progress has been made in the case of suspensions of noninteracting type relatively large particles, and consequently satisfactory methods have evolved which allow the estimation of settling rates and/or their overall rheological behavior. Excellent accounts of the developments in this area are available in the literature, e.g., see Refs. *1–3*.

In contrast to this, much less is known about the analogous problem involving flocculated systems which are inevitably encountered whenever the particles are in the colloidal range. Due to the strong interplay between hydrodynamic and colloidal forces, the degree of flocculation varies from one system to another with additional dependence on the chemical environments, e.g., pH, nature of charge on individual particles, etc. It is therefore evident that the "flow unit" in such systems is a cluster of loosely packed fine particles rather than individual solid particles. Such clusters are known as "flocs." Typical examples wherein this type of behavior is encountered include aqueous suspensions of China clay, coal, TiO_2, etc. Obviously the rates of sedimentation required for designing thickeners and settling tanks and the rheological behavior are influenced by the size and volume fraction of flocs rather than those of the solid particles (*4*). Furthermore, since the flocs are loosely packed clusters, the fluid flow takes place through and around such an ensemble, albeit the latter contribution is likely to dominate the overall macroscopic hydrodynamic behavior of the suspension. Evidently, one must have an adequate model for a floc and a satisfactory understanding of the fluid mechanical aspects before undertaking the modeling of such systems.

One such model, which has gained wide acceptance in recent years (*5–8*), envisions a floc to be an effective composite sphere, i.e., a solid spherical particle embedded in a spherical shell made of isotropic and homogeneous porous medium. This idealization is schematically shown in Fig. 1. Depending upon the value of the permeability of the porous medium, this model does permit flow, howsoever small, through the porous region enclosing the impermeable solid core. Similar hydrodynamic situations also arise in biotechnological applications wherein similar particles are employed as biomass supports (*9*), and in the flow of macromolecular solutions in packed beds where the long chain molecules get adsorbed (*10*) onto the solid particles, thereby offering relatively higher resistance to subsequent fluid flow in the immediate vicinity of each particle. Hence the "effective composite sphere" idealization will be used here for modeling flocs.

It is readily acknowledged that in all the aforementioned applications one usually encounters fluid flow taking place relative to assemblages or clouds of particles rather than a single particle in isolation. Obviously, in the nondilute ranges of concentration, interparticle interactions play an important role in the overall fluid mechanical behavior of flocculated suspensions. Therefore, in addition to the governing equations (momentum and continuity) and the model of an individual floc (as described in the foregoing), a mathematical description of the interparticle interactions is also needed. One such approach, which replaces the many body difficult problem by a conceptually much simpler problem, and which also has had considerable success in providing satisfactory representation of multiparticle systems, is the so-called free surface cell model of Happel (*2*, *11*). For instance, it has been shown that this approach has yielded satisfactory results for the motion of ensembles of bubbles and drops in Newtonian (*12*) as well as in non-Newtonian media (*13–16*), the pressure drop for the flow of Newtonian and non-Newtonian fluids in fixed and fluidized beds (*17–19*), and the settling rates of noninteracting-type suspensions (*20*), while others have employed this model to examine heat and mass transfer aspects in multiparticle systems (*14*, *21*, *22*) and to explain the rheological

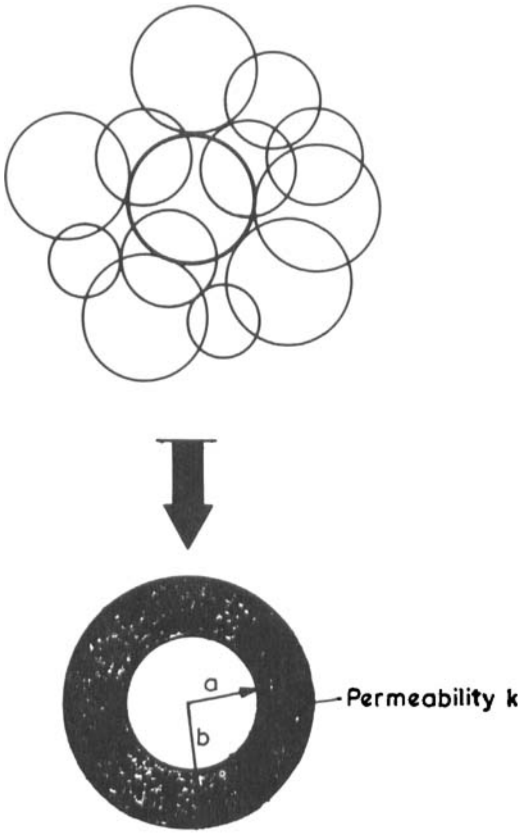

FIG. 1. Idealization of a floc.

behavior of blood (*23*). In view of the success of the free surface cell model in a wide variety of flows, it will be used here in modeling the sedimentation behavior of flocculated suspensions.

In this paper the equations of continuity and of motion have been solved for the creeping flow (Reynolds number below unity) of Newtonian fluids through an assemblage of composite particles (such as shown in Fig. 1), and the sedimentation rates of flocculated suspensions are predicted over wide ranges of conditions. However, the ensuing analysis takes into account only the fluid mechanic aspects of the system.

GOVERNING EQUATIONS AND FORMULATION

Consider the slow (inertialess) steady and isothermal flow of an incompressible Newtonian fluid relative to an assemblage of composite spheres. The free surface cell model of Happel postulates that the effect of neighboring particles on a particle can be adequately accounted for by enclosing it in a hypothetical spherical fluid envelope of radius R_∞ such that the voidage (ϵ) of each cell is equal to the statistically averaged voidage of the entire assemblage. This idealization is shown schematically in Fig. 2. Note that this definition of voidage does not include the contribution arising from the porous shell surrounding each particle.

Let us assume that each composite particle consists of a solid core of

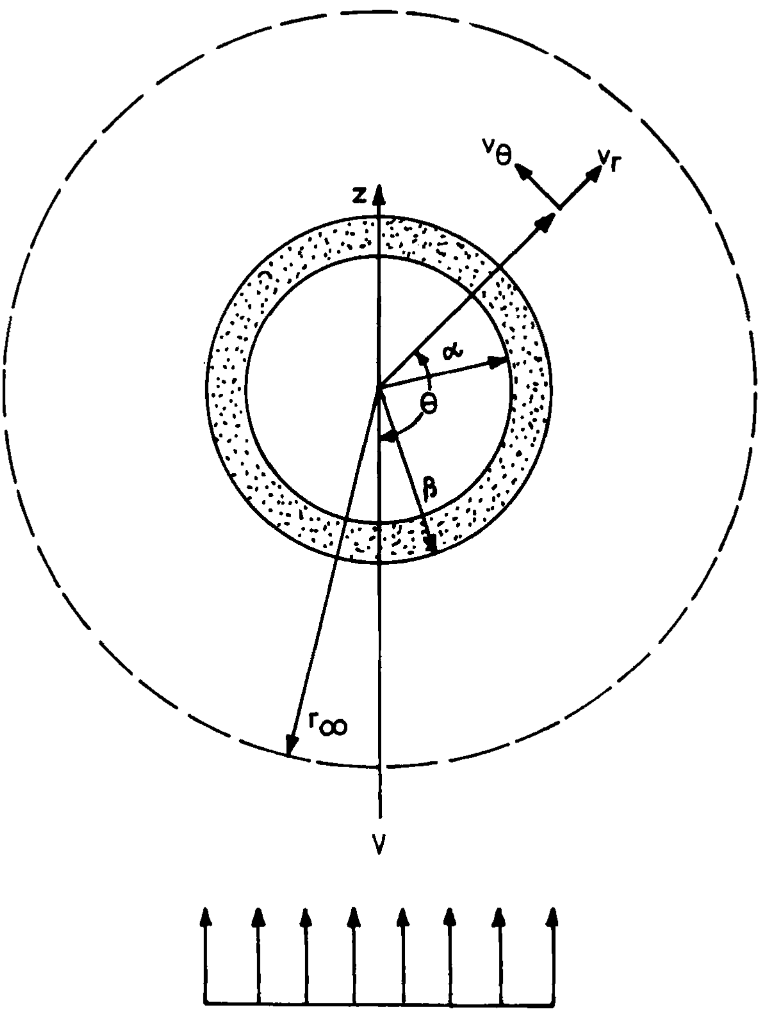

FIG. 2. Schematic representation of the flow configuration.

radius a which is surrounded by a shell of homogeneous and isotropic porous material of permeability k and radius b.

The spherical coordinate system will be used, and owing to ϕ symmetry, the flow is axisymmetric and two dimensional. The flow domain can be divided into two zones as follows.

(1) Free Flowing Region ($b \leq r \leq R_\infty$)

For this zone the equations of continuity and of motion can be written as

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\mu \nabla^2 V = \nabla p \tag{2}$$

(2) Porous Region ($a \leq r \leq b$)

The equations describing flow in this region are not as straightforward as Eqs. (1) and (2), and have been a subject of controversy in the literature. It is now generally recognized that the flow of an incompressible Newtonian fluid through an isotropic and homogeneous material conforms to the so-called Brinkman equation written as follows:

$$-(\mu/k)\mathbf{V}^* + \mu^* \nabla^2 \mathbf{V}^* = \nabla p^* \tag{3}$$

and the equation of continuity can be written in its macroscopic form as

$$\nabla \cdot \mathbf{V}^* = 0 \tag{4}$$

In Eqs. (3) and (4), asterisks refer to a macrosopically averaged quantity relating to the flow region $a \leq r \leq b$. Furthermore, μ^* denotes an effective viscosity in the porous shell region which theoretically can be different from μ, albeit the available limited evidence (*24*) suggests otherwise, i.e., $\mu = \mu^*$. Therefore, here the two viscosity values would be assumed to be equal, and this indeed simplifies the equations appreciably.

Unlike the boundary conditions for flows involving solid boundaries, some confusion exists in the case of flow situations involving porous boundaries (*25*). It is now generally agreed that the physically realistic boundary conditions for this problem are that of no slip at the inner solid particle and of continuity of the velocity and stress components (shear and normal) at the porous interface. Hence the boundary conditions for the present situation can be written as follows.

At $r = a$:

$$v_r^* = 0 \text{ and } v_\theta^* = 0 \text{ (no slip)} \tag{5}$$

At $r = b$:

$$v_r^* = v_r \tag{6a}$$

$$v_\theta^* = v_\theta \tag{6b}$$

$$\tau_{rr}^* = \tau_{rr} \tag{6c}$$

$$\tau_{r\theta}^* = \tau_{r\theta} \tag{6d}$$

At $r = R_\infty$:

$$\tau_{r\theta} = 0 \tag{7a}$$

$$v_r = -V\cos\theta \tag{7b}$$

Equations (7a) and (7b) represent the standard boundary conditions introduced by Happel (*11*), and have been widely used in the literature. As discussed in the preceding section, when the approximation $\mu = \mu^*$ is introduced, Eq. (6c) is tantamount to the continuity of pressures, i.e., $p^* = p$ at $r = b$.

Owing to the axisymmetry, it is customary to introduce a stream function for both flow zones, and thus one can write for $a \le r \le b$:

$$v_r^* = -\frac{1}{r^2 \sin\theta}\frac{\partial \Psi^*}{\partial \theta} \tag{8a}$$

$$v_\theta^* = \frac{1}{r\sin\theta}\frac{\partial \Psi^*}{\partial r} \tag{8b}$$

Similarly, one can introduce analogous stream function for the region $b \le r \le R_\infty$.

With the help of these definitions of velocity components, the equations of motion can be rewritten as

$$E^4\Psi^* - \frac{1}{k}E^2\Psi^* = 0, \qquad a \le r \le b \tag{9a}$$

$$E^4\Psi = 0, \qquad b \le r \le R_\infty \tag{9b}$$

where the differential operator E^2 in spherical coordinates is given by

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) \tag{10}$$

The standard forms of Eqs. (9a) and (9b) suggest that the general solutions are of the following forms, respectively:

$$\Psi^* = -\frac{kV}{2}\left\{\frac{E}{\phi} + F\phi^2 + G\left(\frac{\cosh\phi}{\phi} - \sinh\phi\right) + H\left(\frac{\sinh\phi}{\phi} - \cosh\phi\right)\right\}\sin^2\theta, \qquad \text{for } \alpha \le \phi \le \beta \tag{11}$$

$$\Psi = -\frac{kV}{2}\left\{\frac{A}{\phi} + B\phi + C\phi^2 + D\phi^4\right\}\sin^2\theta, \qquad \text{for } \beta \le \phi \le \phi_\infty \tag{12}$$

where a, b, r, etc. have been made dimensionless with respect to permeability (k) as:

$$\phi = r/\sqrt{k};\ \alpha = a/\sqrt{k};\ \beta = b/\sqrt{k}$$

$$\phi_\infty = R_\infty/\sqrt{k}, \text{ etc.}$$

The general solutions, of Eqs. (9a) and (9b), given by Eqs. (11) and (12), respectively, contain eight unknown constants (A, B, C, ..., H) which are to be evaluated by using the eight boundary conditions expressed by Eqs. (5), (6), and (7), with Eq. (6c) being simply replaced by $p^* = p$. Once the relevant stream functions are evaluated, it is straightforward to calculate the pressure and shear stress at the surface of a particle, which in turn can be integrated to obtain the drag force experienced by an assemblage of given voidage (or concentration) (*26*). The resulting equation for the drag force (F_D) is

$$F_D = \frac{4\pi\mu Vb}{3\beta}\left[3B - \frac{B\beta^2}{\phi_\infty^2} - 10D\phi_\infty\beta^2\right] \tag{13}$$

where the values of B and D are obtained as outlined in the preceding section, and ϕ_∞ is related to the macroscopic voidage (ϵ) or the concentration (C_s) of the assemblage by the relations

$$\phi_\infty = \beta(1 - \epsilon)^{-1/3} = \beta C_s^{-1/3} \tag{14}$$

Although the form of Eq. (13) is convenient for analyzing the results on pressure drop in fixed beds, the quantity of central interest in analyzing the phenomenon of sedimentation is the hindered settling velocity (V), and it can easily be shown (*11*) that Eq. (13) can be rearranged in the form of a ratio of hindered settling velocity (V) to that of a single rigid particle of radius a (V_0) as

$$\frac{V}{V_0} = \frac{F_0}{F_D} = \frac{9}{2}\alpha\left\{\frac{1}{3B - \dfrac{B\beta^2}{\phi_\infty^2} - 10D\phi_\infty\beta^2}\right\} \tag{15}$$

Hence Eq. (15) provides a theoretical framework for the calculation of (V/V_0) as a function of the concentration of a sedimenting suspension and other variables such as properties of the suspending medium (ρ, μ), of the particles (ρ_s, a), and of floc parameters (b, k).

RESULTS AND DISCUSSION

Although it is readily recognized that most colloidal suspensions display varying degrees of flocculation depending upon the chemical environments and other factors (such as pH, shape and charge on particles, etc.), it is really difficult to ascertain the size and permeability of individual flocs. Often indirect methods are employed to infer the values of floc size and concentration, etc. For instance, Firth and Hunter (4) evaluated the average floc size and concentration for a series of suspensions of PMMA, titanium dioxide, kaolinite, and silica from their rheological properties. Based on their calculations, it is safe to assume that the ratio of floc radius to particle radius (β/α) is of the order of 2 to 5. However, no indication of the permeability of a floc is available in the literature. Finally, it is appropriate to add here that though the aforementioned estimates of floc sizes and all other such evaluations are usually model-dependent values, they are assumed here to bear some degree of resemblance with reality. Bearing in mind the above-noted factors, the theoretical results on (V/V_0) have been calculated for the following ranges of conditions:

$$1 < \alpha \leq 5;\ 1.25 \leq (\beta/\alpha) < 12;\ 0 < C_s < 0.70$$

The variation by a factor of 5 in the values of α reflects the corresponding range in the values of permeability whereas (β/α) is a measure of the ratio of floc size to particle size and C_s here refers to the volume concentration of flocs which can be related to the concentration of particles via Eq. (14) for a particular value of (β/α).

Prior to the presentation and discussion of the new results obtained herein, it is appropriate to examine the behavior in a few well-known limiting cases as discussed below:

(1) As $\alpha \rightarrow \beta$, i.e., the suspension consists of nonporous particles of radius *b*, and the results so obtained are in agreement with those available in the literature. Likewise, when $\beta \rightarrow \alpha$, i.e., the present results approach the expected limiting behavior corresponding to rigid particles of radius *a* (*2*).

(2) When the cell boundary (R_∞) becomes infinitely large, one would expect the flow situation to correspond to that of flow around an isolated composite sphere. The drag expression given by Eq. (13) indeed reduces to the results available in the literature for such a flow situation (*8*).

Thus, the analysis presented herein includes several known results as limiting cases.

Figures 3 to 9 show typical dependence of (V/V_0) on floc concentration and the value of β for a fixed value of α in each case. An inspection of these figures shows that, generally, the settling velocity initially decreases as the concentration is increased up to about 30–40% by volume. Beyond this value of concentration, depending upon the values of α and β, the settling velocity either decreases very slowly or remains constant, and shows a weak upward trend. However, when the value of β is not too different from that of α, the velocity ratio monotonically decreases with increasing concentration. This is partly due to the fact that under these conditions the porous shell is almost impermeable and the suspension approaches the behavior of noninteracting type of suspensions.

Intuitively, one would expect the settling behavior of flocculated systems to be governed by the relative magnitudes of the three components of the

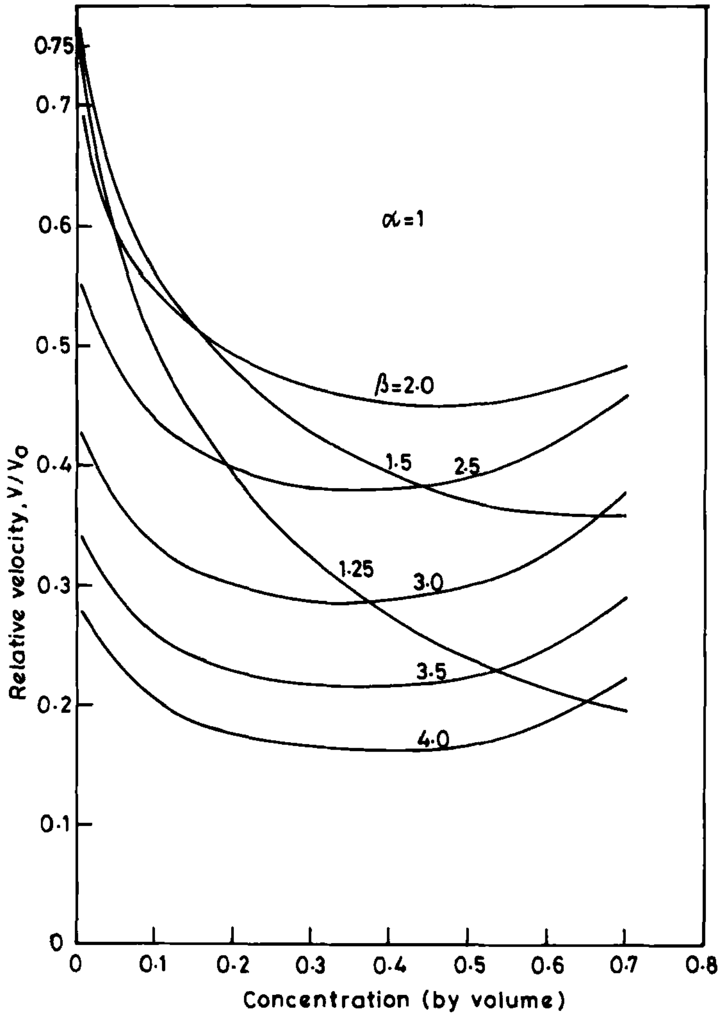

FIG. 3. Velocity ratio (V/V_0) as a function of concentration (C_s) and floc size (β) for $\alpha = 1$.

total resistance to fluid flow: due to solid particles of radius a, due to the porous shell of thickness $(b - a)$, and due to the confining boundary $(\phi = \phi_\infty)$. Owing to intimate connection between the resistance and ease of flow, it is possible to explain the present results in terms of the ease and extent of flow through the porous zone $(\alpha \leq \phi \leq \beta)$ and the free zone $(\beta \leq \phi \leq \phi_\infty)$, respectively. As mentioned above, the general characteristics of the results shown in Figs. 3–9 confirm the presence of the afore-

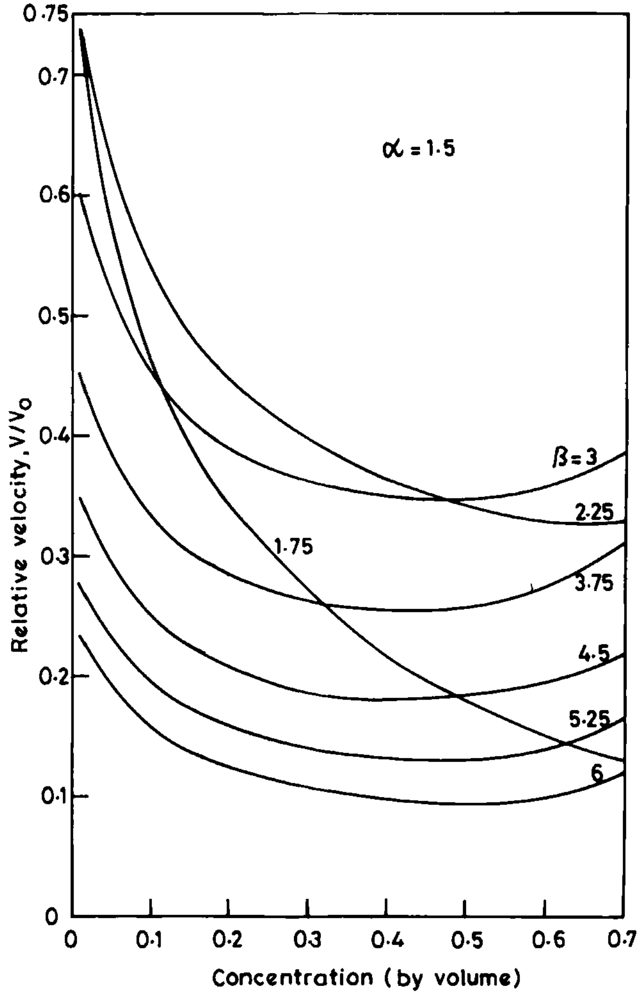

FIG. 4. Velocity ratio (V/V_0) as a function of concentration (C_s) and floc size (β) for $\alpha = 1.5$.

mentioned two competing mechanisms. From a detailed examination of the results in Figs. 3–9, the following salient features can be summarized:

1. By keeping the values of α and β fixed, an increase in the floc concentration causes the settling velocity to decrease. This is a consequence of the reduction in the interparticle distance. However, the rate of reduction of (V/V_0)–C_s curves is not as rapid as in the case of noninteracting-type suspensions. This is simply due to the fact that in the latter case there is no possibility of flow through the particles. Therefore, in this range of

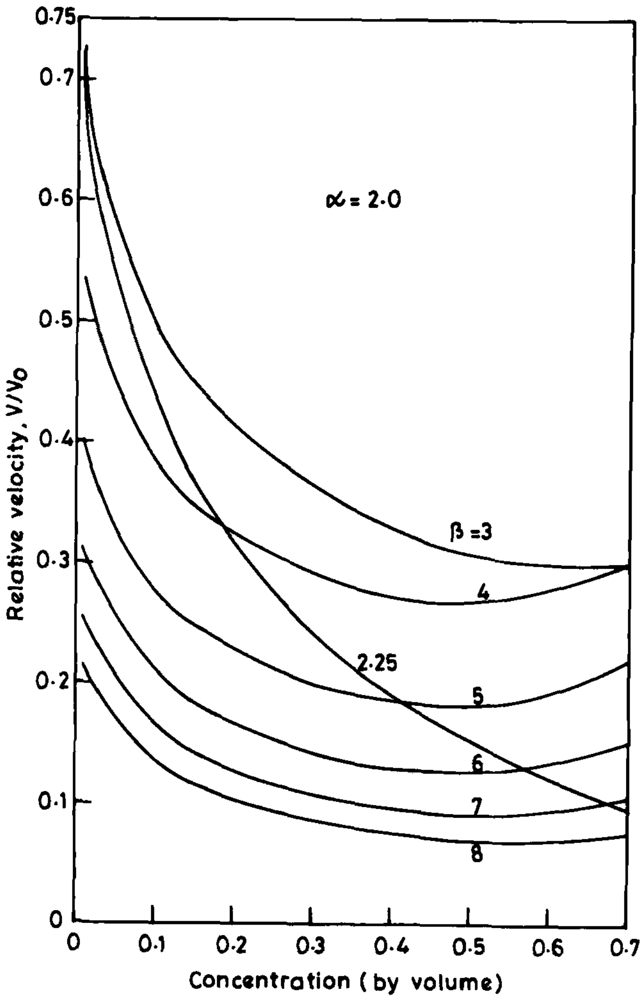

FIG. 5. Velocity ratio (V/V_0) as a function of concentration (C_s) and floc size (β) for α = 2.0.

conditions, the gradual decrease in the interparticle separation dominates over the countereffect caused by a degree of flow through the porous shell. This trend persists up to about $C_s \approx$ 30–40%.

A further increase in concentration accompanied by a concomitant decrease in free flowing area causes an increasing amount of fluid flow to take place through the porous shell, thereby counterbalancing the increase in drag force due to decreasing interparticle distance. This manifests itself as a nearly flat part of the curves in Figs. 3 to 9. Obviously, this phenomenon

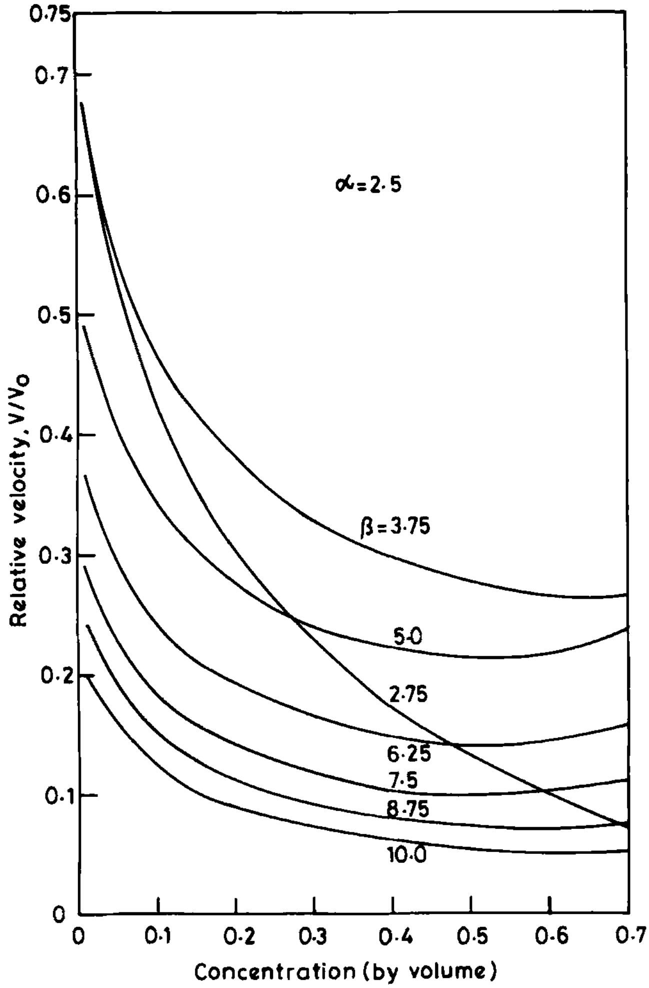

FIG. 6. Velocity ratio (V/V_0) as a function of concentration (C_s) and floc size (β) for $\alpha = 2.5$.

will be extended over wider values of concentration when β/α is high. As long as $\beta/\alpha \leqslant 2$ or so, this effect is seen to be absent, and for $\beta/\alpha > 2$ it is present under the complete range of conditions studied herein. With a further increase in concentration, a major portion of the flow takes place through the porous shell, which results in a decrease in the overall drag force, thereby causing an increase in the value of (V/V_0). For instance, in the case of a 70% concentrated suspension, $\phi_\infty = 1.12\beta$, and if β/α is large (say of the order of 4–5), it is evident that the $\alpha \leq \phi \leq \beta$ region would

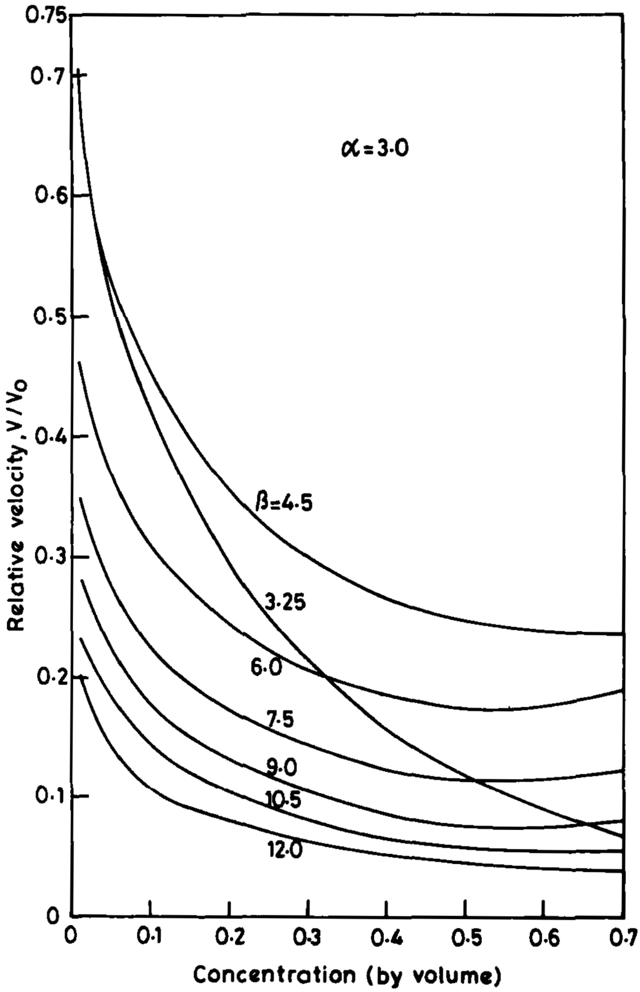

FIG. 7. Velocity ratio (V/V_0) as a function of concentration (C_s) and floc size (β) for $\alpha = 3$.

be much greater than the $\beta \leq \phi < \phi_\infty$ region. This is in contrast to the behavior observed in the case of noninteracting-type suspensions.

2. One would expect the permeability of the porous shell to play a significant role in determining the sedimentation behavior of flocculated suspensions. Although the permeability (k) does not enter the analysis explicitly, its effect can be studied by examining the variation of sedimentation rates with α for given values of β and C_s. For a constant particle size (a), an increase in the value of α is clearly associated with the material

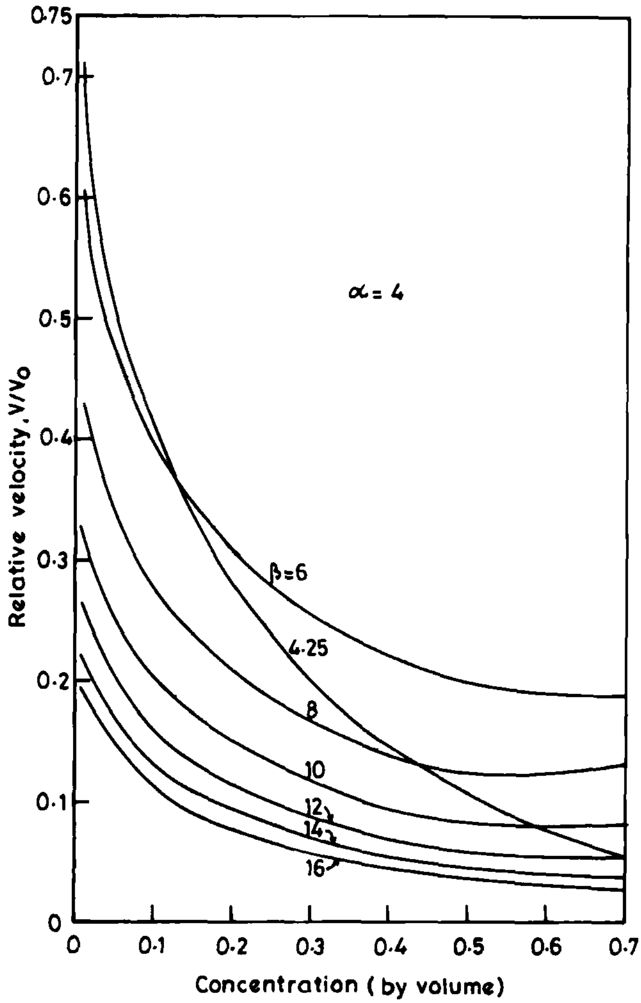

FIG. 8. Velocity ratio (V/V_0) as a function of concentration (C_s) and floc size (β) for α = 4.

becoming more and more impermeable (i.e., approaching solidlike behavior). This in turn would be tantamount to an increased resistance in the region $\alpha \leq \phi \leq \beta$, and the settling velocity must drop under all conditions. Indeed, the results shown in Figs. 3–9 corroborate this assertion.

Alternately, one can also elucidate the effect of changing permeability by varying the value of β under otherwise constant conditions. Hence, for given values of C_s and α, an increase in the value of β reflects a decrease in permeability, and this would again mean the sedimentation rates fall

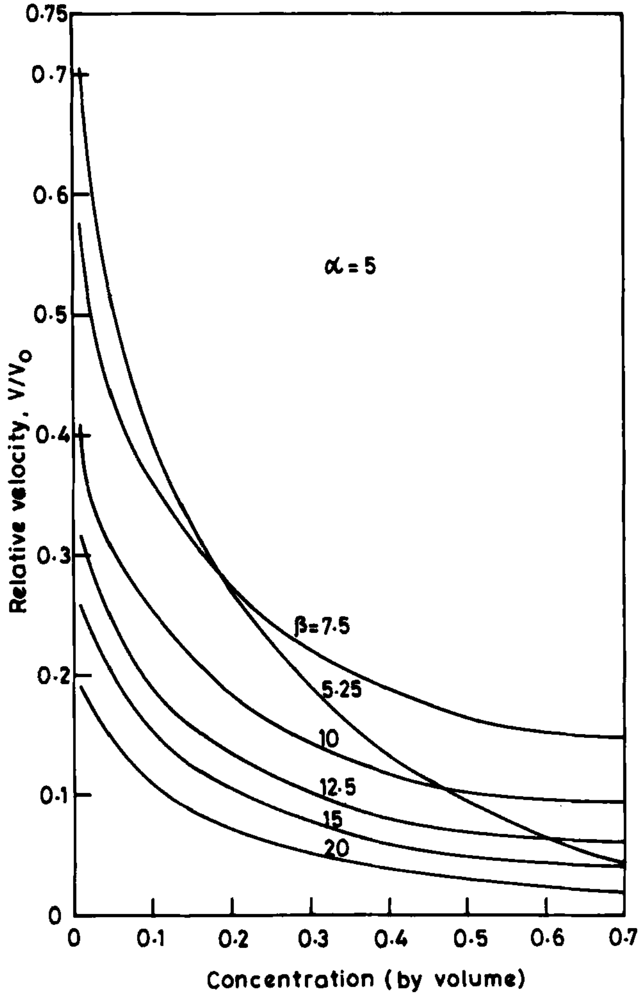

FIG. 9. Velocity ratio (V/V_0) as a function of concentration (C_t) and floc size (β) for $\alpha = 5$.

on account of increased resistance. This fact is also borne out by the results shown in Figs. 3–9.

To recap, the analysis presented herein permits the estimation of sedimentation rates for flocculated suspensions. For a low floc/particle size ratio, and at low concentration, the settling velocity falls with increasing concentration, albeit the rate of drop is *lower* than that for the noninteracting-type suspensions. Above a critical value of concentration, (V/V_0) virtually remains constant and shows a weak upward turn only after the concentration has exceeded a critical value. From a practical viewpoint, the results reported herein are useful for sizing settling tanks and thickeners. In order to employ the present theoretical results, knowledge of the following variables is needed: particle size (a), floc size (b), permeability (k), concentration (C_s), and the properties of the suspending medium. Of these, floc size and concentration can be inferred from rheological measurements as has been outlined by Firth and Hunter (*4*).

CONCLUSIONS

In this work, flocs have been modeled as an effective porous sphere, i.e., a solid sphere embedded in a shell of isotropic and homogeneous porous medium. Based on this idealization, the equations of continuity and motion have been solved analytically in conjunction with Happel's free surface cell model. This analysis enables the calculation of the rates of sedimentation of flocculated suspensions, as required for the sizing of thickeners and settling tanks. The results have been expressed in the form of a dimensionless velocity ratio as a function of dimensionless particle and floc size, and the floc concentration. The effect of the permeability of flocs is implicitly taken into account via the dimensionless particle and floc size. The results reported herein encompass wide ranges of conditions. For low permeability systems, the rates of sedimentation are similar to those for noninteracting-type suspensions. Initially, the settling rate decreases with increasing concentration, followed by a narrow region where the velocity ratio is virtually insensitive to changes in concentration, and finally it shows a weak upturn. The present study can be used in conjunction with rheological properties for formulating a systematic strategy for the design of thickeners and settling tanks to handle colloidal systems.

NOMENCLATURE

$A, B, \ldots, H$	constants appearing in stream functions, Eqs. (11) and (12) (—)
a	radius of solid sphere (m)
b	radius of floc (m)
C_s	floc concentration (by volume) (—)
F_d	drag force on the assemblage (N)
k	permeability of the porous shell (m^2)
p	pressure (Pa)
r	radial coordinate (m)
R_∞	cell boundary (m)
V	sedimentation velocity (m/s)
$\mathbf{V}$	velocity vector (m/s)

v_r, v_θ	r and θ components of $\mathbf{V}$ (m/s)
$\alpha = a/\sqrt{k}$	dimensionless particle radius (—)
$\beta = b/\sqrt{k}$	dimensionless floc radius (—)
$\phi = r/\sqrt{k}$	dimensionless radial coordinate (—)
$\phi_\infty = R_\infty/\sqrt{k}$	dimensionless cell boundary (—)
θ	angular coordinate (—)
μ	viscosity of fluid (Pa · s)
τ_{rr}, $\tau_{r\theta}$	components of stress tensor (Pa)
Ψ	stream function (m^3/s)

Superscript

*	refers to an average value of a quantity relating to porous region

Subscript

0	refers to a quantity pertaining to a single sphere

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